

Supplementary Material: Evidence for an Irreducible κ -like Structure in Galactic Dynamics and Lensing Phenomenology

Pengjie Huang

February 8, 2026

Supplementary Material

S1. Robustness of the κ -like Collapse

This section provides robustness tests for the κ -like collapse reported in Section II of the main text. No new conclusions are introduced here; all figures serve to verify that the observed collapse behavior is insensitive to methodological choices such as binning strategy, subsample selection, and dynamical definition.

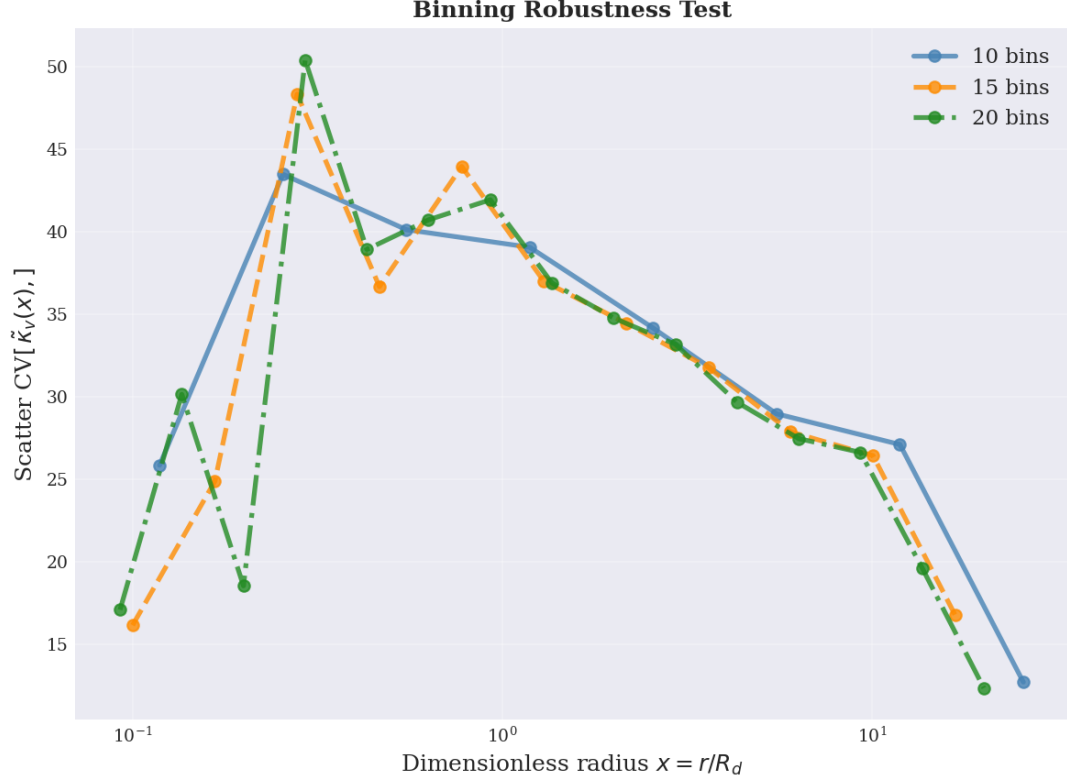


Figure 1: Coefficient of variation (CV) of $\tilde{\kappa}(x)$ computed using 10, 15, and 20 logarithmic bins. All binning choices produce qualitatively similar CV patterns, demonstrating that the observed collapse is not an artifact of binning strategy. This figure supports the robustness of the collapse reported in Section II.

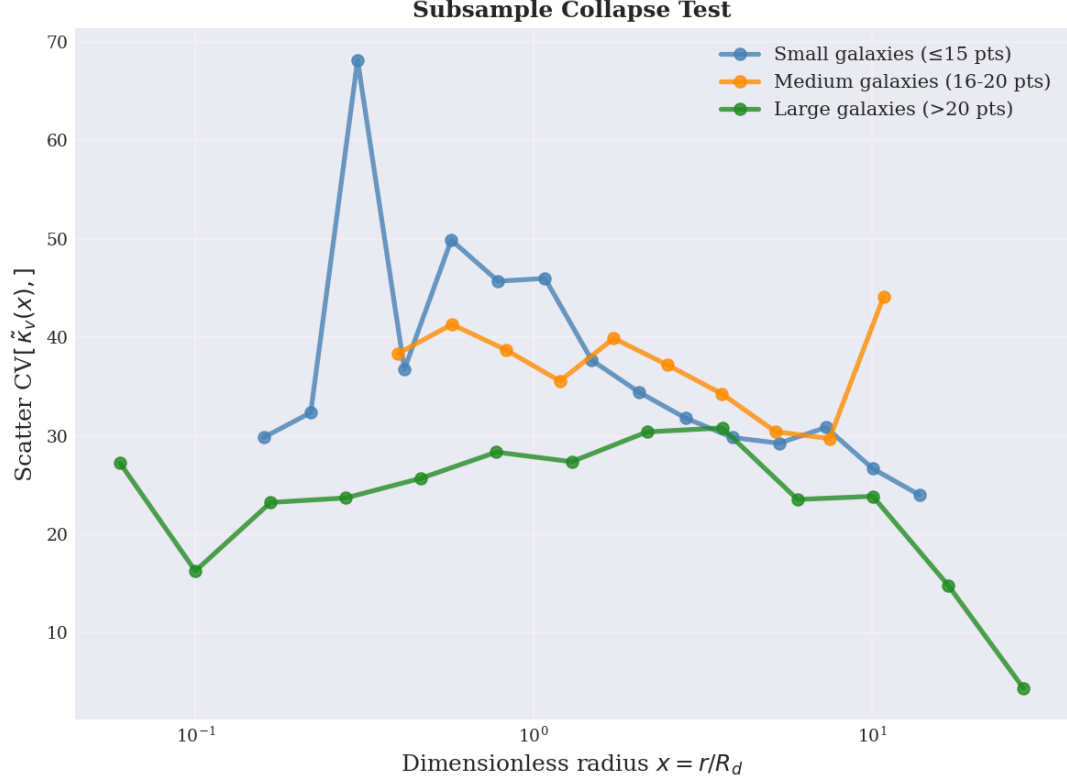


Figure 2: Collapse behavior for different galaxy subsamples divided by the number of available data points. All subsamples exhibit similar CV patterns, indicating that the observed κ -like structure is not driven by a particular galaxy class or by outliers.

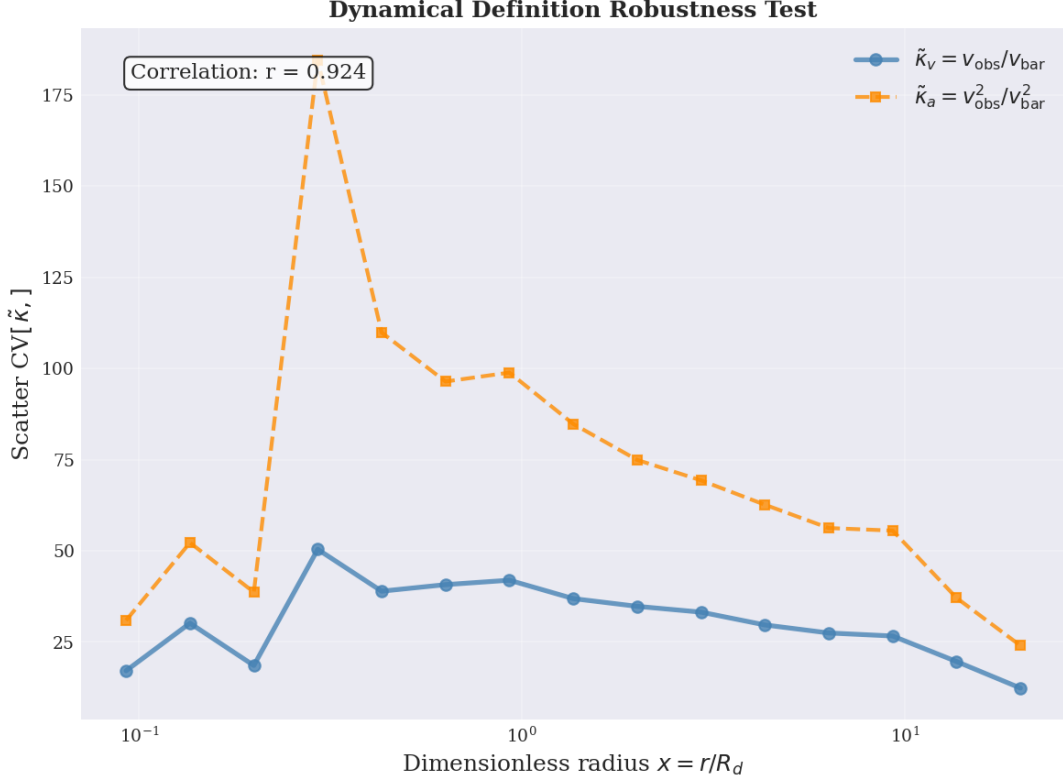


Figure 3: Comparison of collapse behavior under two alternative dynamical definitions: $\tilde{\kappa}_v = v_{\text{obs}}/v_{\text{bar}}$ and $\tilde{\kappa}_a = v_{\text{obs}}^2/v_{\text{bar}}^2$. The high correlation between the resulting CV profiles demonstrates that the collapse is insensitive to the specific dynamical observable employed.

S2. Details of Mass Reparameterization Tests

This section provides additional details supporting the mass irreducibility results discussed in Section III.A of the main text. The figures below demonstrate that physically reasonable mass reparameterizations do not eliminate the κ -like collapse and, in fact, typically enhance it.

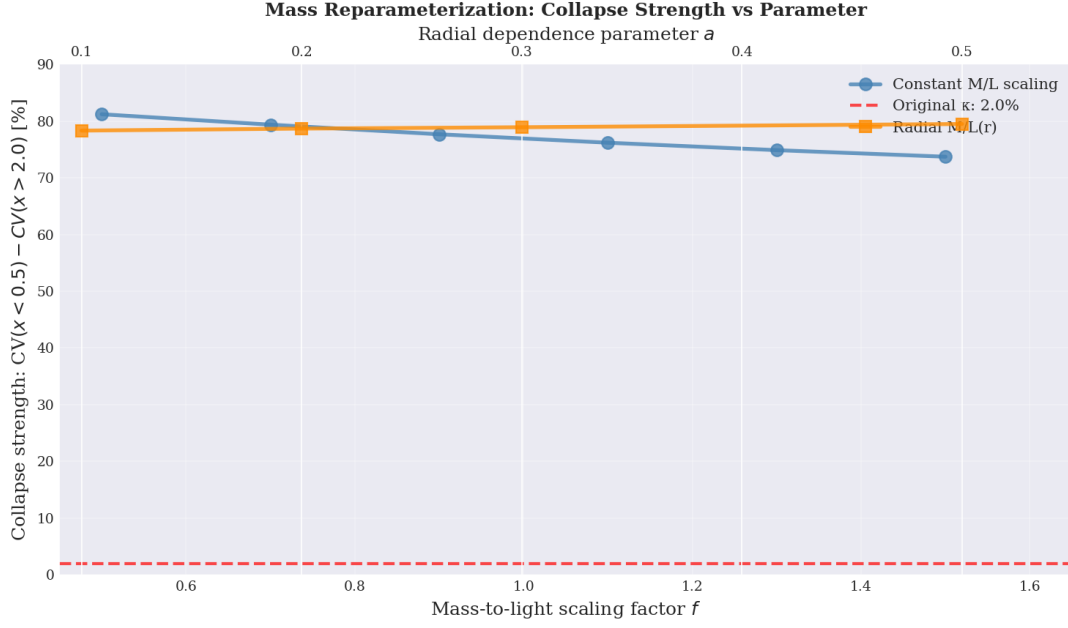


Figure 4: Collapse strength as a function of mass reparameterization. Both constant mass-to-light scaling and radial mass profiles increase the collapse strength relative to the original data. This confirms that mass reparameterization cannot eliminate the observed κ -like structure.

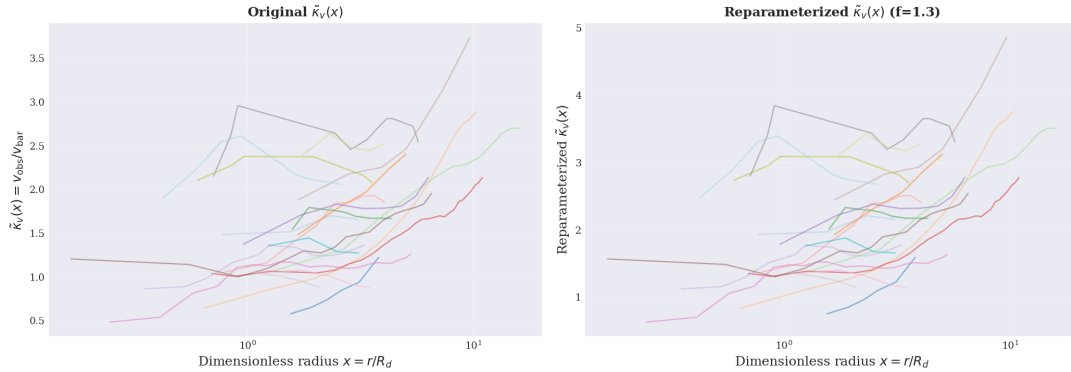


Figure 5: Overlay of original and mass-reparameterized $\tilde{\kappa}(x)$ profiles for representative galaxies. Mass scaling shifts profiles uniformly but preserves their relative structure and collapse behavior, illustrating visually why mass reparameterization fails to remove the κ -like structure.

S3. Comparison with Halo and MOND-like Models

This section provides supplementary comparisons between the κ -like structure and halo-based or MOND-like modeling approaches, corresponding to Sections III.B and III.C of the main text. These figures demonstrate that while such models can reproduce compa-

rable scatter, they do not eliminate the underlying structure.

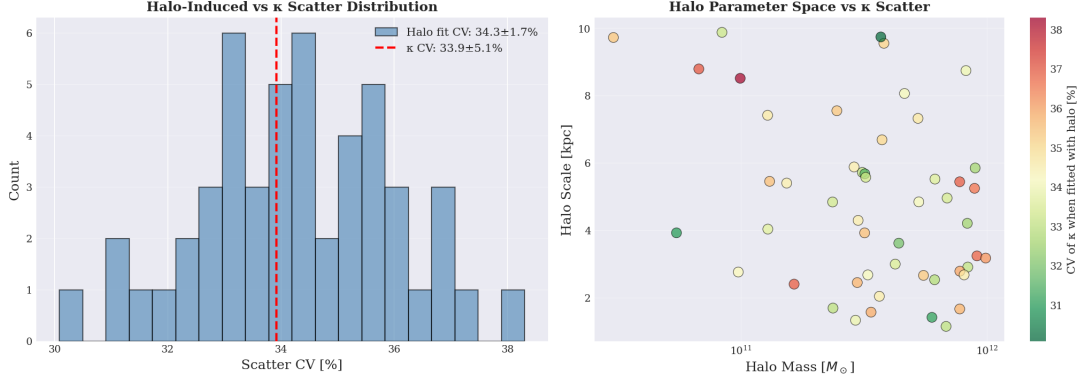


Figure 6: Comparison between scatter induced by halo profile fitting and the observed κ -like scatter. The comparable CV levels indicate that halo models absorb, rather than eliminate, the underlying structure.

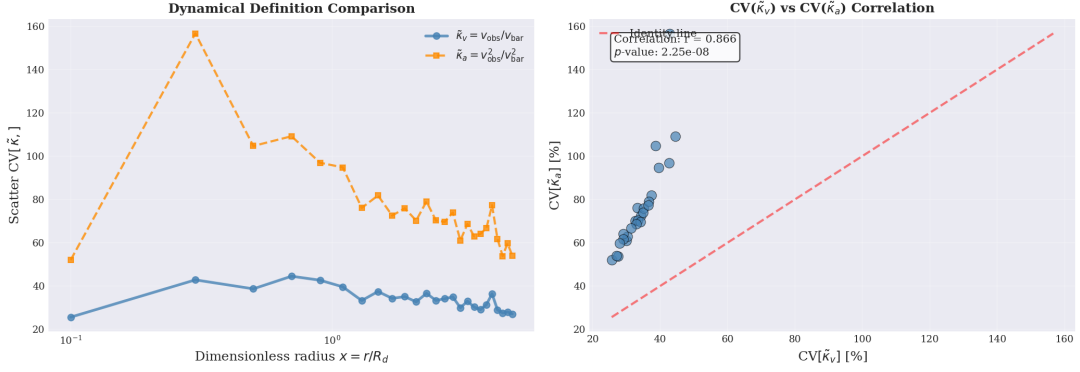


Figure 7: Collapse behavior under MOND-like velocity-based and acceleration-based definitions. Both definitions exhibit similar collapse patterns, confirming that the structure is not tied to a specific dynamical formulation.